

A FREQUENCY DOMAIN TLM METHOD

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ABSTRACT

A new TLM method is presented which operates directly in the frequency domain while following the basic time domain solution procedure. The new method combines the flexibility of the conventional TLM method with the computational efficiency of frequency domain techniques. S-parameters for 3D waveguide discontinuities are calculated and good agreement is found with other methods.

1. INTRODUCTION

Time domain methods are known as computationally intensive general purpose field simulators suitable to analyze transmission line structures and discontinuities of arbitrary geometry[1][2]. While these methods are very helpful in understanding the behavior of fields and waves at circuit discontinuities without writing special purpose programs, time domain methods are not very effective tools in the design of frequency selective microwave circuits. This is so because time domain methods, by virtue of the impulse excitation carry the entire frequency spectrum through the algorithm and only after a lengthy processing time and the Fourier transform of the resulting impulse response, the frequency spectrum of interest can be selected.

In this paper we introduce a new method that uses, in principle, the time domain procedure of the TLM method, but for only one frequency per computation run, and thus avoids the processing of unnecessary frequency information. The computational efficiency of this new frequency domain TLM(FDTLM) method is comparable if not superior over that of other frequency domain methods, but, in addition, retains the flexibility known from the time domain TLM(TDTLM) method to characterize arbitrary shaped circuit structures.

This new approach is best understood by recalling that the time domain TLM method establishes an equivalence between the electromagnetic field in space and the impulse distribution at the junctions (node) of the transmission line network. This method simulates the realistic evolution of the impulse distribution with time. The general TLM algorithm is comprised of events relating the incident and reflected impulses at each time step at each node:

$$V_k^r = S \cdot V_k^i \quad (1)$$

The reflected impulses become incident impulses at the adjacent transmission line at the next time step:

$$V_{k+1}^i = C \cdot V_k^r \quad (2)$$

where S and C are the scattering and connection matrices.

By substituting (1) into (2), the scattering event (1) and the transmission event (2) can be combined into:

$$\begin{aligned} V_{k+1}^i &= A \cdot V_k^i & k=0,1,2,3,\dots \\ A &= C \cdot S \end{aligned} \quad (3)$$

Obviously, equation (3) is a simple iteration procedure for solving a system of linear equations with a coefficient matrix A and initial values V_0^i [3]. Therefore, the TLM algorithm is essentially a physical simulation of the simple mathematical iteration procedure of (3). Since this procedure is computationally very slow, also the TDTLM is of poor computational efficiency, which can only be improved by using parallel processors or special programming techniques.

On the other hand, looking at the TLM algorithm in the context of solving a system of linear equations, one is attempted to utilize advanced techniques developed in linear algebra, instead of simple iteration method, to improve the

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computational efficiency of the TLM. This is the focus of this paper. Several concepts developed here lead to a new TLM algorithm which establishes a direct relationship between the solution procedure in the time and the frequency domain, and at the same time combines the flexibility of the time domain TLM with the computational efficiency of frequency domain methods.

2. EXCITATION

The differences between time and frequency domain methods is the excitation: Time domain methods work with impulses while frequency domain methods use sinusoidal waves. In the new TLM method, we use a novel excitation which is not impulse nor sinusoidal wave but a combination of both: An impulse sequence with its magnitude modulated by a sinusoidal wave. This waveform can also be regarded a continuous sinusoidal wave sampled at discrete times (Fig.1). At any time step this new excitation retains the form of an impulse but the modulated amplitude envelope of the series of impulses contains the information of the structure analyzed. This approach allows one to transform the TLM solution procedure directly into the frequency domain. In comparison to the conventional TLM method, this new technique obtains the information only at one frequency during one run of computation. However, as the solution procedure is essentially carried out in the frequency domain, numerous advanced frequency domain techniques can be readily implemented to greatly enhance the efficiency of the method. The computer resources required in this new method are comparable to other less flexible frequency domain methods while the ability to analyze arbitrarily shaped guide wave structures, known from the conventional TLM method, is still preserved.

3. INTRINSIC SCATTERING MATRIX

Considering a space discretized by the TLM network with N exterior branches connecting the space to the surrounding space (Fig.2). At these exterior branches, incident impulses, with their magnitude modulated by a sinusoidal wave as described before, are injected and the reflected impulses are observed. These reflected impulses, after a sufficiently long period of time, would become a modulated impulse sequence with the same modulation frequency as the incident ones.

The magnitude of the reflected impulses will be related linearly to that of the incident impulses since there are no non-linear events taking place in the TLM network. Therefore, the magnitudes of the incident v^i and reflected v^r impulses at the exterior branches are related by the following relationship:

$$v^r = M \cdot v^i \quad (4)$$

M is defined as the intrinsic scattering matrix of the structure which is solely determined by the properties of the structure itself and the modulation frequency and can be derived from its scattering matrix S and connection matrix C . Normalizing the voltages by the branch impedances :

$$u = Y^{\frac{1}{2}} \cdot v \quad (5)$$

where $Y^{\frac{1}{2}}$ is a diagonal matrix with the i th element being $Y_i^{\frac{1}{2}}$; Y_i is the admittance of the i th branch. Substituting (5) into (4), we obtain:

$$u^r = Y^{-\frac{1}{2}} \cdot M \cdot Y^{\frac{1}{2}} \cdot u^i = m \cdot u^i \quad (6)$$

It is easy to show that, according to the reciprocity theorem [4], the normalized intrinsic scattering matrix m is symmetrical.

4. ALGORITHM

The intrinsic scattering matrix described above fully characterizes the structure analyzed and plays the center role in the new FDTLM method. After the intrinsic scattering matrix has been found, all the properties of the structure, such as the propagation constant Γ of the guided wave structure attached to the discontinuity and the scattering parameters of the 3-D discontinuity itself, can be readily computed. The diaoptics technique can be easily implemented in the new algorithm, simply by dividing the whole structure into several sub-structure. Finding the intrinsic scattering matrices for each of substructures and combining them by simple matrix operations, leads to the intrinsic scattering matrix of the whole structure. A detailed description of the algorithm is quite lengthy and will not be given here. In the following, we only give a brief description for the 2-D guiding structure.

For the guiding structure the intrinsic scattering

matrix m is constructed for a slice of waveguide which contains one node in the propagation direction (Fig.3). In this case, we rewrite equation (6) as follows:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (7)$$

where b_1, b_2 stand for the reflected voltage vectors which contain the reflected voltages of the left and right branches, respectively. a_1, a_2 denote the incident voltage vectors. By performing the following variable transformation in equation (7)

$$\begin{aligned} v_1 &= (a_1 + b_1); & i_1 &= (a_1 - b_1) \\ v_2 &= (a_2 + b_2); & i_2 &= (a_2 - b_2) \end{aligned} \quad (8)$$

we obtain:

$$\begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} \quad (10)$$

Here i_1, i_2 represent the total currents, v_1, v_2 the total voltages. The submatrices A, B, C, D can be expressed with the matrices $m_{11}, m_{12}, m_{21}, m_{22}$.

For the mode supported in the structure, the voltage v_2 and current $-i_2$ differ from v_1 and i_1 by only a constant factor $\exp(-\Gamma\Delta z)$. With Δz being the mesh size parameter in the propagation direction. Now equation (10) can be rewritten as:

$$e^{-\Gamma\Delta z} \cdot \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} \quad (11)$$

which can be further simplified into:

$$\cosh(\Gamma\Delta z) \cdot v_1 = A \cdot v_1 \quad (12)$$

Equation (12) is the standard form of an eigenvalue problem which can be solved by, for instance, the QR factorization method. Let the i th eigenvalue of (11) be Γ_i and its corresponding eigenvector be v_i , then the corresponding total current can be obtained by

$$i_i = -\sinh(\Gamma_i\Delta z) \cdot C \cdot v_i \quad (13)$$

Finally, the incident and reflected impulses of the i th mode, a_i and b_i , can be obtained from equation (8) and from there the s-parameters are determined.

5. NUMERICAL RESULTS

Various calculations and comparisons for both 2-D and 3-D problems have been made to validate this new approach. Fig.4 shows the results of S_{11} for a dielectric obstacle of finite length in a rectangular waveguide [5]. Fig.5 shows the results of S_{11} and S_{12} of the microstrip step discontinuity [6]. The results are in good agreement. The CPU time is about 5 minutes in the first case and 30 minutes in the second case, both on a SUN SPARC STATION 2. This is considerably faster than what the conventional TDTLM method can achieve analyzing the same structure on the same serial machine.

6. CONCLUSION

We have introduced a new TLM method which operates directly in the frequency domain while following the basic time domain solution procedure. The new method combines the flexibility of the conventional TLM method with the computational efficiency of frequency domain techniques. Several new concepts leading to this new method have been introduced. Numerical results of s-parameters for 3D waveguide discontinuity have been presented.

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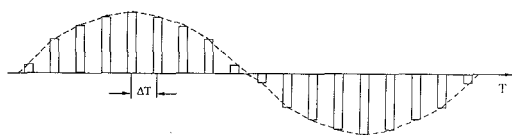


Fig.1 Impulse sequence with its magnitude modulated by a sinusoidal wave

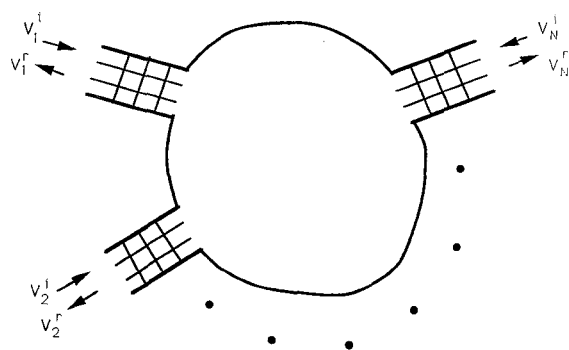


Fig.2. A space discretized by the TLM network with N exterior branches connecting the space to the surrounding space

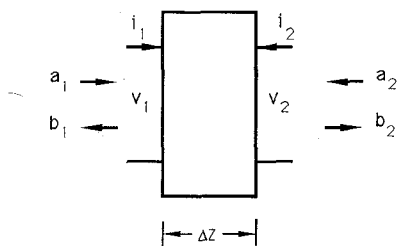


Fig.3. A slice of waveguide with a length of Δz in z direction

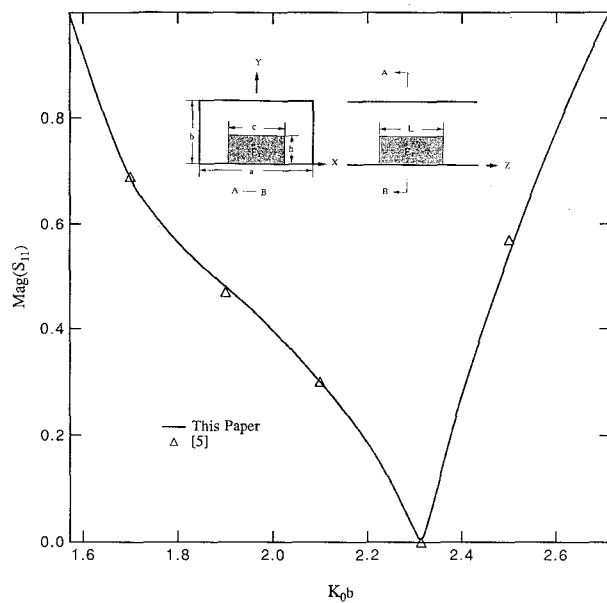


Fig.4 The reflection coefficient for a dielectric obstacle of finite length in a rectangular waveguide ($a=2b$, $c=0.888b$, $h=0.399b$, $l=0.8b$, $\epsilon_r=6$)

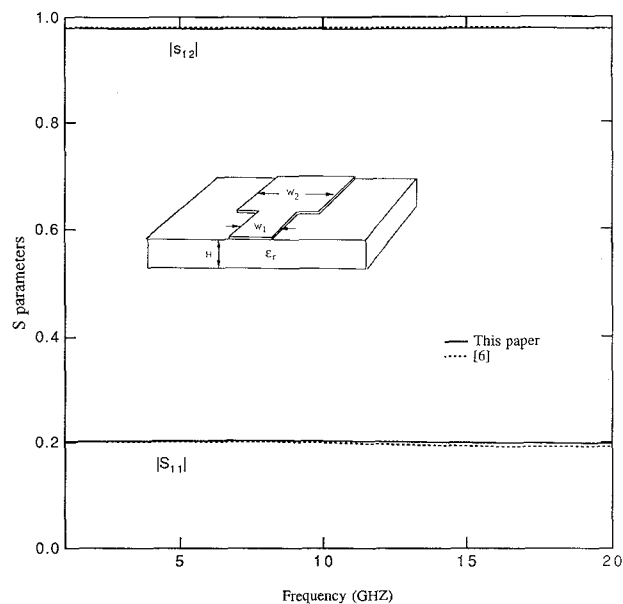


Fig.5 Frequency-dependent S parameters of the microstrip step-in-width ($w_1=0.6\text{mm}$, $w_2=2w_1$, $h=0.6\text{mm}$, $\epsilon_r=10$)